Towards a PDL Framework for Reasoning about Causality

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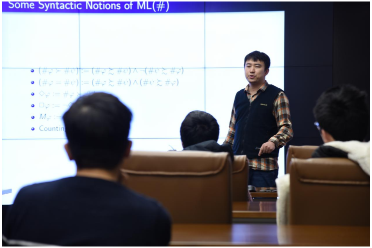
Joint work with



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Outline

Causality

- Causality explains how one event leads to another, forming the backbone of our understanding of change and prediction.
- Philosophical studies from Hume and Kant to modern analyses have long examined how and why things happen. Cf. an overview by De Pierris & Friedman (2024)
- In today's data-driven era, large datasets offer new opportunities to apply and test causal theories through systematic interventions and outcome analysis.

Previous Models

- **Counterfactual Approach** – David Lewis (1973)
  - Causality analyzed through hypothetical alternatives (counterfactuals)
- **Intervention-Based Approach** – Halpern & Pearl (Pearl 2000; Halpern & Pearl 2005; Halpern 2016)
  - Uses structural equations to model causal relationships between variables
  - Worlds are described by variables with specific values
  - Structural equations are interpreted via interventions—inherently dynamic

Example from HP

We want to determine whether a forest fire was caused by lightning or an arsonist. The situation is modeled using three variables:

- FF for Forest fire: $FF = 1$ if there is a fire, $FF = 0$ otherwise
- L for Lightning: $L = 1$ if lightning occurred, $L = 0$ otherwise
- MD for Match dropped (by arsonist): $MD = 1$ if a lit match was dropped, $MD = 0$ otherwise

The causal relationship is captured by the structural equation:

$$FF = \max(L, MD)$$

This means the forest fire occurs if either lightning or a match drop happens. For example, if $MD = 0$ and $L = 1$, then $FF = 1$.

Our Motivation

- In expressions like “L = 1,” L is a syntactic variable, while 1 represents semantic content.
- Halpern (2016, pp. 10–12) notes a close connection with propositional logic: “I often identify binary variables with primitive propositions in propositional logic.”
- Interventions are inherently dynamic. The dynamic nature of interventions aligns naturally with Propositional Dynamic Logic (PDL), which models how actions/programs change truth values across states.

Question: *If we take propositional dynamic logic (PDL) as the starting point for studying causality, what can we achieve?*

Our Admissions Example

Consider an undergraduate student, Alice, who is applying for graduate school. The set of proposition letters is Prop $= \{p_1, p_2, \ldots, p_8\}$, where:

- p_i $(i = 1, 2)$: Alice scores well on the i-th TOEFL test.
- p_3, p_4, p_5: Alice has a good GPA, publication record, and reference letter, respectively.
- p_6: The graduate school receives a good TOEFL score, i.e., p_6 is true iff p_1 or p_2 is true.
- p_7: The graduate school evaluates Alice's academic materials positively, i.e., p_7 is true iff at least two of p_3, p_4, p_5 are true.
- p_8: The graduate school enrolls Alice, i.e., p_8 is true iff both p_6 and p_7 are true.

The Dependence Graph

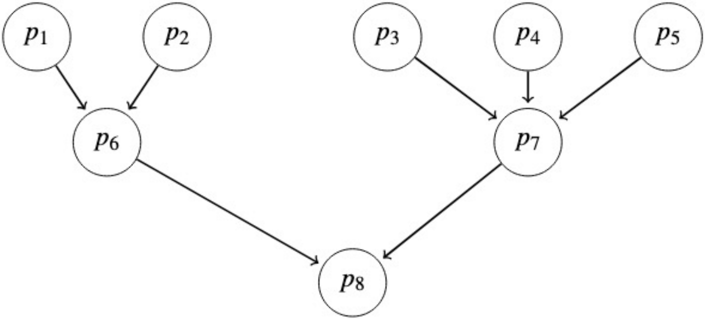

Figure: Dependence Graph

Dependence Structure

Definition 1

Given a finite set Prop of propositional letters, a Prop-dependence structure is a tuple $\mathbb{D} = (\mathsf{Prop}, <, f)$ such that

- $<$ is the dependence relation, an acyclic binary relation on Prop, where $p < q$ means that the value of q depends on p.
- The dependence function is a partial function f from Prop to the set Form of all classical propositional formulas built from Prop. If $\{q \in \mathsf{Prop} \mid q < p\} \neq \emptyset$, then $f(p)$ is a Boolean combination of proposition letters in $\{q \in \mathsf{Prop} \mid q < p\}$; otherwise, $f(p)$ is undefined.

Intuitively, $f(p)$ describes how the value of p is determined by the values of the proposition letters in $\{q \in \mathsf{Prop} \mid q < p\}$. We denote the domain of f as $Dom(f)$ and $\mathsf{Prop} - Dom(f)$ as $Bas(\mathbb{D})$, i.e. the basic propositions.

Example Continued

In our example, p_6 only depends on p_1 and p_2, p_7 only depends on p_3, p_4 and p_5, and p_8 only depends on p_6 and p_7. The formulas $f(p_6), f(p_7), f(p_8)$ are:

$$f(p_6) = p_1 \vee p_2,$$
$$f(p_7) = (p_3 \wedge p_4) \vee (p_3 \wedge p_5) \vee (p_4 \wedge p_5),$$
$$f(p_8) = p_6 \wedge p_7.$$

Order Function

- In dependence structures, $<$ being acyclic ensures $<$-minimal propositions which do not depend on other propositions
- Order function: inductive structure on Prop induced from the dependence relation

Definition 2

Given a Prop-dependence structure $\mathbb{D} = (\mathsf{Prop}, <, f)$, the order function $o_{\mathbb{D}} : \mathsf{Prop} \to \mathbb{N}$ is defined as follows:

$$o_{\mathbb{D}}(p) := \begin{cases} 0 & \text{if } \{q \in \mathsf{Prop} \mid q < p\} = \emptyset \\ max\{o_{\mathbb{D}}(q) \mid q < p\} + 1 & \text{otherwise.} \end{cases}$$

We say that $o_{\mathbb{D}}(p)$ is the order of p.

In the example, $o_{\mathbb{D}}(p_i) = 0$ for $i = 1, 2, 3, 4, 5$, $o_{\mathbb{D}}(p_6) = o_{\mathbb{D}}(p_7) = 1$ and $o_{\mathbb{D}}(p_8) = 2$.

Basic Propositions

- A situation is a set of propositions, each true or false.
- The truth of propositions in a situation is based on its associated dependence structure $\mathbb{D} = (\text{Prop}, <, f)$:
  - Propositions of order 0 are basic: true or false independently of others.
  - Propositions of higher order depend on the propositions they are related to, i.e., the set $\{q \in \text{Prop} \mid q < p\}$, and their truth values are computed using $f(p)$
- The dependence function f reflects the direct dependence of p on $\{q \in \text{Prop} \mid q < p\}$ that p directly depends on
- Based on the dependence function, we can compute how p depends on basic propositions indirectly

Composite Dependence Function

Definition 3

Given any dependence structure $\mathbb{D} = (\text{Prop}, <, f)$, the composite dependence function $g : \text{Prop} \to \text{Form}$ is a total function such that:

- If $o(p) = 0$, then $g(p) = p$.
- If $o(p) > 0$, then $g(p) = f(p)[g(q_1)/q_1, \ldots, g(q_n)/q_n]$, where $q_1, \ldots, q_n$ enumerate all proposition letters in $\{q \in \text{Prop} \mid q < p\}$ and $g(p)$ is obtained from $f(p)$ by uniformly substituting q_i by $g(q_i)$, $i = 1, \ldots, n$.

Example 4

In the example, $g(p_i) = p_i$ for $i = 1, 2, 3, 4, 5$,
$g(p_6) = f(p_6) = p_1 \vee p_2$, $g(p_7) = (p_3 \wedge p_4) \vee (p_3 \wedge p_5) \vee (p_4 \wedge p_5)$,
$g(p_8) = (p_1 \vee p_2) \wedge ((p_3 \wedge p_4) \vee (p_3 \wedge p_5) \vee (p_4 \wedge p_5))$.

Situation

A situation is fully determined by the truth values of basic propositions and the dependence function f.

Definition 5

Given a Prop-dependence structure $\mathbb{D}$, a $\mathbb{D}$-situation is $w = (\mathbb{D}, T_w)$, where $T_w \subseteq Bas(\mathbb{D})$ is the set of true basic propositions.

Example 6

In the admission example, consider the dependence structure $\mathbb{D}$, the set of basic propositions is given by $Bas(\mathbb{D}) = \{p_1, p_2, p_3, p_4, p_5\}$. For instance, $w_1 = (\mathbb{D}, \{p_1\})$, $w_2 = (\mathbb{D}, \{p_1, p_2\})$, $w_3 = (\mathbb{D}, \{p_1, p_2, p_4\})$ are situations.

Dynamic Events

- Situations are not static: they can be changed by events
- Consider three kinds of events for p:
  - $+p$: asserting p to be true
  - $-p$: retracting p to be false
  - $\downarrow p$: removing the dependence of p from other propositions without changing its truth value

Formal Definition of Events

Definition 7

Let $\mathbb{D} = (\text{Prop}, <, f)$ be a Prop-dependence structure and $w = (\mathbb{D}, T_w)$ be a $\mathbb{D}$-situation.

(i) The assertion event $+p$, the retraction event $-p$ and the dependence removal event $\downarrow p$ on $\mathbb{D} = (\text{Prop}, <, f)$ all result in the new dependence structure $\mathbb{D}_p = (\text{Prop}, <_p, f_p)$ such that

- $<_p = < -\{(q, p) \mid q \in \text{Prop}\}$,
- f_p is obtained from f by making $f(p)$ undefined.

(ii) Re. assertion event $+p$, the $\mathbb{D}$-situation w is updated to the $\mathbb{D}_p$-situation $w_{+p} = (\mathbb{D}_p, T_w^{+p})$, where $T_w^{+p} = T_w \cup \{p\}$.

(iii) Re. retraction event $-p$, the $\mathbb{D}$-situation w is updated to the $\mathbb{D}_p$-situation $w_{-p} = (\mathbb{D}_p, T_w^{-p})$, where $T_w^{-p} = T_w - \{p\}$.

(iv) Re. dependence removal event $\downarrow p$, the $\mathbb{D}$-situation w is updated to the $\mathbb{D}_p$-situation $w_{\downarrow p} = (\mathbb{D}_p, T_w^{\downarrow p})$, where $T_w^{\downarrow p} = T_w \cup \{p\}$ if p is true at w and $T_w^{\downarrow p} = T_w - \{p\}$ if p is false at w.

Universal Model

Definition 8

Given any finite set Prop of propositions and any Prop-dependence structure $\mathbb{D}$, the $\mathbb{D}$-universal model $\mathbb{M}$ is the smallest set containing all $\mathbb{D}$-situations and is closed under taking assertion, retraction and dependence removal events.

Universal model: all situations in a dependence structure $\mathbb{D}$

Includes both original $\mathbb{D}$-situations and those reached via finite event sequences.

Causal Propositional Dynamic Logic CPDL: Syntax

Given a finite set Prop of proposition letters, the formulas of the CPDL are defined as follows:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid [+p]\varphi \mid [-p]\varphi \mid [\downarrow p]\varphi$$

where $p \in$ Prop.

- The logical connectives $\bot, \top, \rightarrow, \vee, \leftrightarrow$ are defined in the standard way
- $[+P]\varphi$, $[-P]\varphi$ and $[\downarrow P]\varphi$: abbreviations for $[+p_1]\ldots[+p_n]\varphi$, $[-p_1]\ldots[-p_n]\varphi$ and $[\downarrow p_1]\ldots[\downarrow p_n]\varphi$, respectively, where $p_1,\ldots,p_n$ enumerate all propositions in P
- $\langle +p\rangle$, $\langle -p\rangle$ and $\langle\downarrow p\rangle$: abbreviations for $\neg[+p]\neg$, $\neg[-p]\neg$, $\neg[\downarrow p]\neg$, respectively
- Prop(φ): the set of proposition letters occurring in φ

Semantics of CPDL

Definition 9

Given any finite set Prop of proposition letters and any Prop-dependence structure $\mathbb{D}$, the $\mathbb{D}$-universal model $\mathbb{M}$, any situation $w = (\mathbb{D}_w, T_w) \in \mathbb{M}$, the satisfaction relation is defined as follows:

For any $p \in \text{Prop}$:

If $o_{\mathbb{D}_w}(p) = 0$, then $w \Vdash p$ iff $p \in T_w$.
Otherwise, $w \Vdash p$ iff $w \Vdash f_{\mathbb{D}_w}(p)$.

For the Boolean connectives and constants, as usual.

For $[+p]\varphi$, $w \Vdash [+p]\varphi$ iff $w_{+p} \Vdash \varphi$,

For $[-p]\varphi$, $w \Vdash [-p]\varphi$ iff $w_{-p} \Vdash \varphi$,

For $[\downarrow p]\varphi$, $w \Vdash [\downarrow p]\varphi$ iff $w_{\downarrow p} \Vdash \varphi$.

Validity

Definition 10

Given a finite set Prop of proposition letters, we say that a formula φ built up from proposition letters in Prop is Prop-valid, if for any Prop-dependence structure $\mathbb{D}$, any $\mathbb{D}$-universal model $\mathbb{M}$ and any situation $w \in \mathbb{M}$, we have $w \Vdash \varphi$.

We say that φ is valid, if for any finite set Prop containing all the proposition letters occurring in φ, φ is Prop-valid.

Remark: for the definition of validity, Prop may contain more proposition letters than those in φ. Connection with *Ceteris Paribus*.

Some Validities

Proposition 11

The following formulas are valid:

- $[\heartsuit p_1][\heartsuit p_2]\dots[\heartsuit p_n]\varphi \leftrightarrow [\heartsuit p_{i_1}]\,[\heartsuit p_{i_2}]\dots[\heartsuit p_{i_n}]\varphi$, *where* $(i_1, i_2, \dots, i_n)$ *is a re-ordering of* $(1, 2, \dots, n)$ *and* $\heartsuit \in \{+, -, \downarrow\}$.
- $[+p][-p]\varphi \leftrightarrow [-p]\varphi$ *and* $[-p][+p]\varphi \leftrightarrow [+p]\varphi$.
- $[+p][\downarrow p]\varphi \leftrightarrow [+p]\varphi$ *and* $[-p][\downarrow p]\varphi \leftrightarrow [-p]\varphi$.
- $[\downarrow p]\varphi \leftrightarrow (p \wedge [+p]\varphi) \vee (\neg p \wedge [-p]\varphi)$.

A PDL Perspective

The PDL programs we consider are defined as follows:

$$\pi ::= a \mid \pi;\pi \mid \pi \cup \pi \mid \varphi?$$

Below are some validities from the perspective of PDL:

$[\downarrow p]\varphi \leftrightarrow [(p?;+p) \cup (\neg p?;-p)]\varphi$

$[+p_1;+p_2;\ldots;+p_n]\varphi \leftrightarrow [+p_{i_1};+p_{i_2};\ldots;+p_{i_n}]\varphi$.

$[-p_1;-p_2;\ldots;-p_n]\varphi \leftrightarrow [-p_{i_1};-p_{i_2};\ldots;-p_{i_n}]\varphi$.

$[\downarrow p_1;\downarrow p_2;\ldots;\downarrow p_n]\varphi \leftrightarrow [\downarrow p_{i_1};\downarrow p_{i_2};\ldots;\downarrow p_{i_n}]\varphi$.

$[+p;-p]\varphi \leftrightarrow [-p]\varphi$ and $[-p;+p]\varphi \leftrightarrow [+p]\varphi$.

$[+p;\downarrow p]\varphi \leftrightarrow [+p]\varphi$ and $[-p;\downarrow p]\varphi \leftrightarrow [-p]\varphi$.

Adding the Epistemic Dimension

- Extend formulas with two operators: $K\varphi$ (knowledge) and $[\varphi!]\varphi$ (public announcement)
- Satisfaction is evaluated at (E, w), where $w \in \mathbb{M}$ is the actual situation and $E \subseteq \mathbb{M}$ is the epistemic state with $w \in E$
- E captures the situations the agent considers possible
- Event operators modify the physical world—i.e., dependence structures and situation propositions
- Announcement operators update the agent's epistemic state without altering the world

Satisfaction for Epistemic CPDL

- For proposition letters and Boolean combinations, satisfaction is defined as in CPDL, but with reference point (E, w)
- For other formulas, satisfaction is defined as follows:

  $(E, w) \Vdash [+p]\varphi$ iff $(E_{+p}, w_{+p}) \Vdash \varphi$, where
  $E_{+p} = \{v_{+p} \mid v \in E\}$

  $(E, w) \Vdash [-p]\varphi$ iff $(E_{-p}, w_{-p}) \Vdash \varphi$, where
  $E_{-p} = \{v_{-p} \mid v \in E\}$

  $(E, w) \Vdash [\downarrow p]\varphi$ iff $(E_{\downarrow p}, w_{\downarrow p}) \Vdash \varphi$, where $E_{\downarrow p} = \{v_{\downarrow p} \mid v \in E\}$

  $(E, w) \Vdash K\varphi$ iff $(E, w') \Vdash \varphi$ for all $w' \in E$

  $(E, w) \Vdash [\varphi!]\psi$ iff $(E, w) \Vdash \varphi$ implies $(E_{\varphi!}, w) \Vdash \psi$, where
  $E_{\varphi!} = \{v \in E \mid (E, v) \Vdash \varphi\}$

- Assumption: the agent is aware of the events that take place.

More Validities

Proposition 12

The following formulas are valid, where $*, *_i \in \{+, -\}$ *and* $\heartsuit \in \{\epsilon, \neg\}$ *where* ϵ *is the empty string:*

$$K[*p]\varphi \leftrightarrow [*p]K\varphi.$$

$$[\varphi!][*_1 p_1]\ldots[*_n p_n]\heartsuit q \leftrightarrow (\varphi \rightarrow [*_1 p_1]\ldots[*_n p_n]\heartsuit q).$$

Example(continued)

Consider four possible situations regarding Alice's TOEFL test scores:

w_1: Alice scores well on both the first (p_1) and the second (p_2) TOEFL tests.

w_2: Alice scores well on the first test (p_1) but not on the second (p_2).

w_3: Alice does not score well on the first test (p_1) but does on the second (p_2).

w_4: Alice does not score well on either the first (p_1) or the second (p_2) test.

These situations form the epistemic state $E = \{w_1, w_2, w_3, w_4\}$. We now have the following:

> $(E, w_1) \Vdash [p_1!]Kp_6$, i.e. after announcing that Alice gets a good score in the first TOEFL test, she knows that the graduate school will receive a good TOEFL score for sure.
>
> $(E, w_1) \Vdash \neg K[-p_1]p_6 \wedge \neg K[-p_1]\neg p_6$, i.e. Alice does not know for certain whether, if her first TOEFL score were counterfactually revised to be low ($[-p_1]$), the graduate school would receive a good TOEFL score (p_6) or not.

Decidability

Theorem 13

Fix a finite set Prop *of proposition letters. It is decidable whether a formula* φ *is* Prop*-valid or not.*

Proof: by enumeration.

An Abbreviation

The abbreviation $p \rightsquigarrow q$ for $p \neq q \in$ Prop is defined as follows:

$$\bigvee_{\substack{P_1 \cap P_2 = \emptyset \\ P_1 \cup P_2 = \text{Prop} - \{p,q\}}} ([+P_1 \cup \{p\}][-P_2]q \wedge [+P_1][-P_2 \cup \{p\}]\neg q) \vee ([+P_1 \cup \{p\}][-P_2]\neg q \wedge [+P_1][-P_2 \cup \{p\}]q).$$

Intuitively, $p \rightsquigarrow q$ reads 'p has a direct causal effect on q', i.e. there is a possibility that keeping all the other propositions in P_1 and P_2 invariant, changing the truth value of p will change the truth value of q.

Axiomatization: Epistemic CPDL

(1) All instances of propositional tautologies.

(2) Event Axioms

$[\vec{*p}]\neg\varphi \leftrightarrow \neg[\vec{*p}]\varphi$

$[\vec{*p}](\varphi \wedge \psi) \leftrightarrow [\vec{*p}]\varphi \wedge [\vec{*p}]\psi$

$[\vec{*p}]\heartsuit_1 q_1 \wedge [\vec{*p}]\heartsuit_2 q_2 \rightarrow [\vec{*p}][\heartsuit_1 q_1]\heartsuit_2 q_2$

$[*q][\vec{*p}] * q$ where q is not in $\vec{p}$

$[\vec{*p}][*q]\heartsuit r \wedge [\vec{*p}][\heartsuit r] * q \rightarrow [\vec{*p}] * q$ for $q \neq r$

$(p_0 \rightsquigarrow p_1) \wedge \ldots (p_{n-1} \rightsquigarrow p_n) \wedge (p_n \rightsquigarrow p_0) \rightarrow \bot$ where $n \geq 1$

$[\downarrow p]\varphi \leftrightarrow (p \wedge [+p]\varphi) \vee (\neg p \wedge [-p]\varphi)$

$[*p][\heartsuit p]\varphi \leftrightarrow [\heartsuit p]\varphi$

$[*p][\heartsuit q]\varphi \leftrightarrow [\heartsuit q][*p]\varphi$ for $p \neq q$

$(\bigwedge_{r \in \mathsf{Prop}-\{q\}} \neg(r \rightsquigarrow q)) \rightarrow (q \leftrightarrow [\vec{*p}]q)$ where q is not in $\vec{p}$

$*, \heartsuit \in \{+, -\}$ or $\{\epsilon, \neg\}$ where ϵ is the empty string, depending on the positions of $*, \heartsuit$, and we use $[\vec{*p}]$ to denote $[*_1 p_1] \ldots [*_n p_n]$, where $n \geq 0$

Axiomatization

(3) Epistemic axioms:

$K(\varphi \to \psi) \to (K\varphi \to K\psi)$.
$K\varphi \to \varphi$.
$K\varphi \to KK\varphi$.
$\neg K\varphi \to K\neg K\varphi$.

(4) Interaction axiom: $K[*p]\varphi \leftrightarrow [*p]K\varphi$.

(5) Reduction axioms for public announcement:

$[\varphi!][\vec{*}\vec{p}]\heartsuit q \leftrightarrow (\varphi \to [\vec{*}\vec{p}]\heartsuit q)$.
$[\varphi!](\psi \land \theta) \leftrightarrow [\varphi!]\psi \land [\varphi!]\theta$.
$[\varphi!]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi!]\psi)$.
$[\varphi!]K\psi \leftrightarrow (\varphi \to K(\varphi \to [\varphi!]\psi))$.

Rules:

(MP) From $\varphi \to \psi$ and φ infer ψ.

(N) From φ infer $K\varphi$.

(RE) From $\psi \leftrightarrow \theta$ infer $\varphi \leftrightarrow \varphi[\psi/\theta]$, where $\varphi[\psi/\theta]$ is obtained by replacing some occurrences of θ with ψ (here we do not consider the occurrences of θ in the event operators).

Soundness and Completeness

Theorem

Fix a finite set Prop of proposition letters, the system above is sound and complete with respect to all Prop-dependence structure $\mathbb{D}$-based universal models. That is to say, a Prop-formula is derivable in the system above iff it is Prop-valid.

Proof (sketch):

We first reduce a given consistent Prop-formula φ into normal form built up from $[+P][-Q]p$ by applying $\neg, \wedge, K$.

Then we consider the set Δ of all normal form formulas, and the set X of all maximally consistent subset of Δ. We will show that any $\Gamma \in X$ can lead to some (E_Γ, w_Γ) such that $(E_\Gamma, w_\Gamma) \Vdash \Gamma$.

Finally, we can extend the normal form of φ into a maximal consistent subset in X, hence φ is satisfiable.

Comparison with HP's Work: Similarities

HP's Variable-Based Model	**Our Proposition-Based Model**
Variable X with domain $\mathcal{R}(X)$	Proposition letter p_X
Primitive Event: $X = x$	Proposition p_X or its negation $\neg p_X$
Boolean Formula: $(X = x) \wedge (Y = y)$	$(\neg)p_X \wedge (\neg)p_Y$
Context $\vec{u} \in \mathcal{R}(\mathcal{U})$	Truth assignment to all p_U for $U \in \mathcal{U}$
Situation $(M, \vec{u})$	World in a Kripke-style model $(\mathcal{M}, w)$
Intervention $M_{\vec{X} \leftarrow \vec{x}}$	Model update $\mathcal{M}_{*p}$ by event $*p$ $(* \in \{+, -\})$
Intervention formula $[\vec{X} \leftarrow \vec{x}]\,\varphi$	$[*p_X]\,\varphi$ $(* \in \{+, -\})$
$(M, \vec{u}) \Vdash X = x$	$\mathcal{M}, w \vDash (\neg)p_X$
$(M, \vec{u}) \Vdash [\vec{X} \leftarrow \vec{x}](Y = y)$	$\mathcal{M}, w \vDash [\vec{*}\vec{p}_X](\neg)p_Y$

Table: Translation from HP's causal models to CPDL

Comparison with HP's Work: Differences

HP's Variable-Based Framework	**Our Proposition-Based Framework**
Multi-valued variable	Two-valued proposition
Model-specific syntax	Universal syntax
Actual cause formalized via semantics	Actual cause expressible within syntax
Allows cyclic dependence structure	Only acyclic dependence structure
Intervention on endogenous variables	Events on all propositions

Table: Differences

Other Related Works

- Epistemic causal logic (Xie, 2020)
- Dynamic epistemic logic (Baltag, Moss & Solecki, 1998); van Ditmarsch, van der Hoek, & Kooi 2007; van Benthem 2011)
- Propositional dynamic logic (Harel, 2000). We employ three dynamic operators to model three types of information update.
  - $+p$ and $-p$ are similar to 'ontic updates' in van Ditmarsch and Kooi (2008); Herzig and De Lima (2006), but they also update direct dependence relations represented by the f_p function, akin to relation updates in preference updating, as explored in (van Benthem & Liu, 2007).
  - $\downarrow p$ represents a direct relation update. On the epistemic side, our treatment mirrors the public announcement operator in (Baltag, Moss & Solecki, 1998) to update an agent's epistemic state, but we also handle iterated events.

Conclusion

- Introduced CPDL, a PDL-based framework for causal reasoning, and its epistemic extension combining causality and knowledge.
- Developed a complete and decidable logical system supporting dynamic updates of facts, dependencies, and epistemic states.
- Compared with the HP framework, highlighting conceptual connections and unique strengths of our approach.

Future Work

- Compare our approach with studies on actual causality (Batusov & Soutchanski 2018; Khan & Lespérance 2021; Khan et al. 2025) based on the situation calculus (McCarthy & Hayes 1981; Reiter 2001).
- Investigate decidability when the language is not fixed, and relate it to the debate on *ceteris paribus* reasoning (Glymour 2002; Woodward 2002; Henschen 2015; Girard & Triplett 2018; Hu 2024).
- Extend the framework to the full power of PDL (including iterations) and explore richer languages for contextualizing causal relations.
- Introduce probabilistic dependence to model non-deterministic outcomes and connect with current AI research.

Thanks

Thank you!

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